FYJC - MATHEMATICS & STATISTICS

HIGHLIGHTS

- ✓ *Solution to all questions*
- ✓ *solutions are put in way the student is expected to reproduce in the exam*
- ✓ *taught in the class room the same way as the solution are put up here . That makes the student to easily go through the solution & prepare him/herself when he/she sits back to revise and recall the topic at any given point of time .*
- ✓ *lastly, if student due to some unavoidable reasons , has missed the lecture , will not have to run here and there to update his/her notes .*
- ✓ *however class room lectures are must for easy passage of understanding & learning the minuest details of the given topic*

PAPER - II

COMBINATIONS

Compiled & Conducted @ JKSC

COMBINATION - QSET1

SUMS ON SELECTION OF COMMITTEE / TEAM

01. there are seven men and three women . Find the number of ways in which a committee of 6 can be formed from these if the committee is to include at least two women **ans : 140**

02. From amongst 8 men and 5 women , a committee of 5 is to be formed so as to include at least 3 women . find the number of ways in which this can be done **ans : 321**

03. a committee of 5 is to be formed out of 6 gents and 4 ladies . In how many ways can this be done if at most two ladies are included **ans : 186**

04. from 4 accountants , 3 lawyers and 5 salesmen , a committee of 7 is to be formed . In how many ways can this be done if it contains at least 4 salesmen **ans : 196**

05. a question paper consists of 11 questions divided into two sections I and II . Section I consists of 5 questions and section II consists of 6 questions . In how many ways can a student select 6 questions taking at least 2 questions from each section **ans : 425**

06. a cricket team of 11 players is to be selected from a group of 15 players out of whom there are 6 are bowlers and 3 are wicket keepers . The team should contain exactly 1 wicket keeper and at least 4 bowlers . Find the number of ways in which this can be done **ans : 198**

07. A committee of 12 persons is to be formed from 9 women and 8 men . In how many ways can this be done if men are in majority **ans : 1134**

08. A student is to answer eight out of 10 questions in an examination .How many choices has he if he must answer at least four out of first five questions **ans : 35**

COMBINATION - QSET2

SUMS ON INCLUDE / EXCLUDE

01. In how many ways can 5 students be selected out of 11 students if

a) 2 particular students are included **ans : 84**

b) 2 particular students are not included **ans : 126**

02. there are 15 players including A , B & C . Find the number of ways in which cricket team of 11 can be chosen if

a) A is already selected as captain **ans:1001**

b) B is injured & is not available **ans : 364**

c) A is selected as captain & at the same time B is not available **ans : 286**

03. The staff of the bank consists of the manager , the deputy manager and 10 other officers . A committee of 4 is to be selected . Find the number of ways in which this can be done so as to

a) include the manager **ans : 165**

b) include the manger but not the deputy manager **ans : 120**

c) neither the manager nor the deputy manager **ans : 210**

04. A student is to answer eight out of 10 questions in an examination .how many choices has he if he must answer the first three questions **ans : 21**

05. in how many ways can 18 objects be divided into 3 groups containing 9 , 6 & 3 objects respectively **ans : $^{18}C_9 \times {}^{9}C_6 \times {}^{3}C_3$**

06. in how many ways can 15 things be divided into 3 groups containing 8 , 4 and 3 things respectively **ans : $^{15}C_8 \times {}^{7}C_4 \times {}^{3}C_3$**

07. from a class of 25 students 10 are to be chosen for a project work . There are 3 students who decide that either all of them will join or none will join . In how many ways can they be chosen . **ans : $^{22}C_{10} + {}^{22}C_7$**

08. a boy has 3 library tickets and 8 books of his interest in the library . Of these 8 books , he does not want to borrow Chemistry part II , unless Chemistry part I is borrowed . In how many ways can he choose three books to be borrowed . **ans : 41**

COMBINATION - QSET3

SUMS ON CHORDS – LINES – TRIANGLES – POLYGONS

01. How many chords can be drawn through 21 points on a circle **ans : 210**

02. Find maximum number of diagonals that can be drawn in n – sided polygon where

1) n = 12 2) n = 15 3) decagon **ans: 54 ; 90 ; 35**

03. Find the number of straight lines obtained by joining 10 points on a plane , if

a) no three points are collinear **ans : 45**

b) four points are collinear **ans : 40**

04. there are 15 points in a plane out of which 5 are collinear . Prove that we can obtain 96 straight lines by joining these points in pairs .

05. Find the number of triangles obtained by joining 10 points on a plane , if

a) no three of them are collinear **ans : 120**

b) four points are collinear **ans : 116**

06. there are 15 points in a plane out of which 5 are collinear . Prove that there are 445 triangles with vertices at these points

07. Each of a set of 5 parallel lines cuts each one of another set of 4 parallel lines . How many different parallelograms can be formed **ans : 60**

08. at the end of meeting , everyone had shaken hands with every one else . It was found that 45 handshakes were exchanged . How many members were present at the meeting . **ans : 10**

EXTRAS

01. in how many ways can a man invite 6 friends to a party so that 2 or more of them remain present . **ans : 57**

02. there are 5 questions in a question paper . In how many ways can a boy solve one or more questions . **ans : 31**

03. in order to pass the examination a minimum is to be secured in each of the 7 subjects . In how many ways can a student fail. **ans : 127**

COMBINATION - QSET4

SUMS ON $^nP_r = {}^nC_r \cdot r!$; $^nC_r = \dfrac{n!}{r!(n-r)!}$

01. $^nP_r = 120$ & $^nC_r = 20$, find n and r **ans :6,3**

02. $^nP_r = 720$ & $^nC_r = 120$, find n and r **ans :10,3**

03. $^nC_6 : {}^{n-3}C_3 = 33 : 4$, find n **ans : 11**

04. $^{14}C_{2r} : {}^{10}C_{2r-4} = 143 : 10$, find r **ans : 4**

05. $^{28}C_{2r} : {}^{24}C_{2r-4} = 225 : 11$, find r **ans : 7**

06. $^{10}C_{r+2} : {}^{10}C_r = 10 : 21$, find r **ans : 5**

07. $^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 20 : 35 : 42$, find n & r **ans: 10,4**

08. $^nC_{r-1} = 495$; $^nC_r = 220$; $^nC_{r+1} = 66$, find n & r **ans: 12,9**

$$\boxed{{}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r}$$

09. $^{14}C_5 + {}^{14}C_6 + {}^{15}C_7 + {}^{16}C_8 = {}^{17}C_x$, find x **ans : 8,9**

10. $^{25}C_4 + {}^{25}C_5 + {}^{26}C_6 + {}^{27}C_7 = {}^{28}C_r$, find r **ans :7,21**

11. $^{12}C_5 + 2.{}^{12}C_4 + {}^{12}C_3 = {}^{14}C_x$, find x **ans : 5,9**

12. $^{47}C_4 + \sum_{r=1}^{5} {}^{52-r}C_3$. **ans : $^{52}C_4$**

13. find the difference between the greatest values of $^{13}C_r$ & $^{11}C_r$ **ans : 1254**

14. find the difference between the greatest values of $^{14}C_r$ & $^{8}C_r$ **ans : 3362**

15. find the difference between the greatest values of $^{15}C_r$ & $^{12}C_r$ **ans : 5511**

COMBINATION – SOLUTION TO QSET-1

01. there are seven men and three women . Find the number of ways in which a committee of 6 can be formed from these if the committee is to include at least two women

7 men , 3 women

committee of 6 (at least 2 women)

Case 1 : Committee contains 4 men & 2 women

This can formed in = $^{7}C_4 \times {}^{3}C_2$. = $^{7}C_3 \times {}^{3}C_1$

= 35 x 3 = 105 ways

Case 2 : Committee contains 3 men & 3 women

This can formed in = $^{7}C_3 \times {}^{3}C_3$. = 35 x 1 = 35 ways

By fundamental principle of ADDITION

Total ways of forming the committee = 140

02. From amongst 8 men and 5 women , a committee of 5 is to be formed so as to include at least 3 women . find the number of ways in which this can be done

8 men , 5 women

committee of 5 (at least 3 women)

Case 1 : Committee contains 2 men & 3 women

This can formed in = $^{8}C_2 \times {}^{5}C_3$. = $^{8}C_2 \times {}^{5}C_2$.

= 28 x 10 = 280 ways

Case 2 : Committee contains 1 men & 4 women

This can formed in = $^{8}C_1 \times {}^{5}C_4$. = $^{8}C_1 \times {}^{5}C_1$.

= 8 x 5 = 40 ways

Case 3 : Committee contains no man & 5 women

This can formed in = $^{5}C_5$. = 1 way

By fundamental principle of ADDITION

Total ways of forming the committee = 321

03. a committee of 5 is to be formed out of 6 gents and 4 ladies . In how many ways can this be done if at most two ladies are included

6 gents and 4 ladies

committee of 5 (at most two ladies)

Case 1 : Committee contains 5 gents & no ladies

This can formed in = ${}^{6}C_5 \times {}^{4}C_0$. = ${}^{6}C_1 \times {}^{4}C_0$.

= 6 x 1 = 6 ways

Case 2 : Committee contains 4 gents & 1 lady

This can formed in = ${}^{6}C_4 \times {}^{4}C_1$. = ${}^{6}C_2 \times {}^{4}C_1$.

= 15 x 4 = 60 ways

Case 3 : Committee contains 3 gents & 2 ladies

This can formed in = ${}^{6}C_3 \times {}^{4}C_2$. = 20 x 6 = 120 ways

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By fundamental principle of ADDITION

Total ways of forming the committee = 186

04. from 4 accountants , 3 lawyers and 5 salesmen , a committee of 7 is to be formed . In how many ways can this be done if it contains at least 4 salesmen

4 accountants , 3 lawyers and 5 salesmen

committee of 7 (at least 4 salesmen)

Case 1 : Committee contains 4 salesmen & 3 others

This can formed in = ${}^{5}C_4 \times {}^{7}C_3$. = ${}^{5}C_1 \times {}^{7}C_3$

= 5 x 35 = 175 ways

Case 2 : Committee contains 5 salesmen & 2 others

This can formed in = ${}^{5}C_5 \times {}^{7}C_2$. = 1 x 21 = 21 ways

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By fundamental principle of ADDITION

Total ways of forming the committee = 196

05. a question paper consists of 11 questions divided into two sections I and II . Section I consists of 5 questions and section II consists of 6 questions . In how many ways can a student select 6 questions taking at least 2 questions from each section

Section I consists of 5 questions and section II consists of 6 questions
student select 6 questions taking at least 2 questions from each section

Case 1 : students selects 2 Q's from Section I & 4 Q's from Section II
This can done in = $^5C_2 \times {}^6C_4$. = $^5C_2 \times {}^6C_2$.
= 10 x 15 = 150 ways

Case 2 : students selects 3 Q's from Section I & 3 Q's from Section II
This can done in = $^5C_3 \times {}^6C_3$. = $^5C_2 \times {}^6C_3$.
= 10 x 20 = 200 ways

Case 3 : students selects 4 Q's from Section I & 2 Q's from Section II
This can done in = $^5C_5 \times {}^6C_2$. = $^5C_1 \times {}^6C_2$.
= 5 x 15 = 75 ways

By fundamental principle of ADDITION
Total ways = 425

06. a cricket team of 11 players is to be selected from a group of 15 players out of whom there are 6 are bowlers and 3 are wicket keepers . The team should contain exactly 1 wicket keeper and at least 4 bowlers . Find the number of ways in which this can be done

6 Bowlers , 3 wicket keepers & 6 batsmen
a cricket team of 11 players is to be selected (exactly 1 wicket keeper & at least 4 bowlers)

Case 1 : Team contains 4 Bowlers , 1 wicket keepers & 6 batsmen
This can formed in = $^6C_4 \times {}^3C_1 \times {}^6C_6$. = $^6C_2 \times {}^3C_1 \times {}^6C_6$
= 15 x 3 x 1 = 45 ways

Case 2 : Team contains 5 Bowlers , 1 wicket keepers & 5 batsmen
This can formed in = $^6C_5 \times {}^3C_1 \times {}^6C_5$. = $^6C_1 \times {}^3C_1 \times {}^6C_1$
= 6 x 3 x 6 = 108 ways

Case 3 : Team contains 6 Bowlers , 1 wicket keepers & 4 batsmen
This can formed in = $^6C_6 \times {}^3C_1 \times {}^6C_4$. = $^6C_6 \times {}^3C_1 \times {}^6C_2$
= 1 x 3 x 15 = 45 ways

By fundamental principle of ADDITION , Total ways = 198

07. A committee of 12 persons is to be formed from 9 women and 8 men . In how many ways can this be done if men are in majority

9 women , 8 men

committee of 12 (men are in majority)

Case 1 : Committee contains 5 women & 7 men

This can formed in = $^{9}C_5 \times {}^{8}C_7$. = $^{9}C_4 \times {}^{8}C_1$.

= 126 x 8 = 1008 ways

Case 2 : Committee contains 4 women & 8 men

This can formed in = $^{9}C_4 \times {}^{8}C_8$. = 126 x 1 = 126 ways

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By fundamental principle of ADDITION

Total ways of forming the committee = 1134

08. A student is to answer eight out of 10 questions in an examination .How many choices has he if he must answer at least four out of first five questions

Case 1 : Student answers 4 Q's from first 5 Q's and 4 Q's from next 5 Q's

This can be done in = $^{5}C_4 \times {}^{5}C_4$ = $^{5}C_1 \times {}^{5}C_1$

= 5 x 5 = 25 ways

Case 2 : Student answers 5 Q's from first 5 Q's and 3 Q's from next 5 Q's

This can be done in = $^{5}C_5 \times {}^{5}C_3$ = $^{5}C_5 \times {}^{5}C_2$

= 1 x 10 = 10 ways

By fundamental principle of addition

Total ways = 45

COMBINATION – SOLUTION TO Q SET-2

01. In how many ways can 5 students be selected out of 11 students if

a) 2 particular students are included

Since 2 particular students are included , the remaining 3 students have to be selected from the remaining 9 students .

This can be done in = $^{9}C_{3}$ = 84 ways

b) 2 particular students are not included

Since 2 particular students are not included , the 5 students have to be selected from the remaining 9 students .

This can be done in = $^{9}C_{5}$ = $^{9}C_{4}$ = 126 ways

02. there are 15 players including A , B & C . Find the number of ways in which cricket team of 11 can be chosen if

a) A is already selected as captain

Since A is already selected as captain , remaining 10 players have to be selected from the remaining 14 players .

This can be done in = $^{14}C_{10}$ = $^{14}C_{4}$ = 1001 ways

b) B is injured & is not available

Since B is injured & is not available , the 11 players have to be selected from the remaining 14 players .

This can be done in = $^{14}C_{11}$ = $^{14}C_{3}$ = 364 ways

c) A is selected as captain & at the same time B is not available

Since A is selected as captain & at the same time B is not available , the remaining 10 players have to be selected from the remaining 13 players .

This can be done in = $^{13}C_{10}$ = $^{13}C_{3}$ = 286 ways

03. The staff of the bank consists of the manager , the deputy manager and 10 other officers . A committee of 4 is to be selected . Find the number of ways in which this can be done so as to

a) include the manager

Since manager is included , remaining 3 members have to be selected from the remaining 11 members (1 deputy manager + 10 officers) .

This can be done in = $^{11}C_3$ = 165 ways

b) include the manger but not the deputy manager

since the manager is included but not the deputy manager , remaining 3 members have to be selected from the remaining 10 officers .

This can be done in = $^{10}C_3$ = 120 ways

c) neither the manager nor the deputy manager

since neither the manager nor the deputy manager is included , the 4 members have to be selected from the remaining 10 officers

This can be done in = $^{10}C_4$ = 210 ways

By fundamental principle of Multiplication ,

Total ways of forming the committee = $^{3}C_2 \times {}^{10}C_3$ = 3 x 120 = 360

04. A student is to answer eight out of 10 questions in an examination .how many choices has he if he must answer the first three questions

since the student must answer first 3 questions , he has to then select the remaining 5 questions from the remaining 7 questions

This can be done in = $^{7}C_5$ = $^{7}C_2$ = 21 ways

05. in how many ways can 18 objects be divided into 3 groups containing 9 , 6 & 3 objects respectively

First 9 objects have to be selected from the 18 objects . This can be done in $^{18}C_9$ ways

Having done that ; next 6 objects have to be selected from remaining 9 objects . This can be done in $^{9}C_6$ ways

Having done that ; last 3 objects have to be selected from the remaining 3 objects . This can be done in $^{3}C_3$ ways . Hence total ways = $^{18}C_9 \times {}^{9}C_6 \times {}^{3}C_3$

06. in how many ways can 15 things be divided into 3 groups containing 8 , 4 and 3 things respectively **ans : $^{15}C_8 \times {}^{7}C_4 \times {}^{3}C_3$**

First 8 things have to be selected from the 15 things . This can be done in $^{15}C_8$ ways

Having done that ; next 4 things have to be selected from remaining 7 things . This can be done in $^{7}C_4$ ways

Having done that ; last 3 things have to be selected from the remaining 3 things . This can be done in $^{3}C_3$ ways .

Hence By Fundamental Principle of Multiplication : total ways = $^{18}C_9 \times {}^{9}C_6 \times {}^{3}C_3$

07. from a class of 25 students 10 are to be chosen for a project work . There are 3 students who decide that either all of them will join or none will join . In how many ways can they be chosen . **ans : $^{22}C_{10} + {}^{22}C_7$**

Case 1 : 3 students decide : all 3 of them will join

Since all 3 students will join , the remaining 7 students have to be selected from the remaining 22 students . This can be done in $^{22}C_7$ ways

Case 2 : 3 students decide : all 3 of them will not join

Since all 3 students will join , the 10 students have to be selected from the remaining 22 students . This can be done in $^{22}C_{10}$ ways

By fundamental principle of addition

Total ways = $^{22}C_7 + {}^{22}C_{10}$

08. a boy has 3 library tickets and 8 books of his interest in the library . Of these 8 books , he does not want to borrow Chemistry part II , unless Chemistry part I is borrowed . In how many ways can he choose three books to be borrowed . **ans : 41**

Case 1 : Chemistry part II is borrowed

Since Chemistry part II is , the boy must have borrowed Chemistry part I . He has to now select remaining 1 books from the remaining 6 books .

This can be done in $^{6}C_1$ = 6 ways

Case 2 : Chemistry part II is NOT borrowed

Since Chemistry part II is NOT borrowed , the boy has to now select 3 books from the remaining 7 books .

This can be done in $^{7}C_3$ = 35 ways Total ways = 41

COMBINATION – SOLUTION TO Q SET-3

01. How many chords can be drawn through 21 points on a circle

2 points on a circle define a chord

$\therefore$ number of chords that can be drawn $= {}^{21}C_2 = 210$

02. Find maximum number of diagonals that can be drawn in n – sided polygon where

1) n = 12 2) n = 15 3) decagon

1) 12 – sided polygon

12 sided polygon

12 points

2 points define a line

$\therefore$ number of line that can be drawn

$= {}^{12}C_2 = 66$

But 12 are sides

$\therefore$ No . of diagonals $= 66 - 12$

$= 54$

2) 15 – sided polygon

15 sided polygon

15 points

2 points define a line

$\therefore$ number of line that can be drawn

$= {}^{15}C_2 = 105$

But 15 are sides

$\therefore$ No . of diagonals $= 105 - 15$

$= 90$

03. Find the number of straight lines obtained by joining 10 points on a plane , if

a) no three points are collinear

10 points

2 points define a line

$\therefore$ number of line that can be drawn $= {}^{10}C_2 = 45$

b) four points are collinear

10 points

2 points define a line

$\therefore$ number of line that can be drawn $= {}^{10}C_2 = 45$

But 4 points are collinear

Number of lines wrongly counted in these 4 collinear points $= {}^{4}C_2 = 6$ instead of 1

Hence

actual lines that can be drawn = 45 – 6 + 1 = 40

04. there are 15 points in a plane out of which 5 are collinear . Prove that we can obtain 96 straight lines by joining these points in pairs .

15 points

2 points define a line

$\therefore$ number of line that can be drawn $= {}^{15}C_2 = 105$

But 5 points are collinear

Number of lines wrongly counted in these 5 collinear points $= {}^{5}C_2 = 10$ instead of 1

Hence actual lines that can be drawn = $105 - 10 + 1 = 96$

05. Find the number of triangles obtained by joining 10 points on a plane , if

a) no three of them are collinear

10 points

3 points define a triangle

$\therefore$ number of triangles that can be drawn $= {}^{10}C_3 = 120$

b) four points are collinear

10 points , 3 points define a triangle

$\therefore$ number of line that can be drawn $= {}^{10}C_3 = 120$

But 4 points are collinear

Number of triangles wrongly counted in these 4 collinear points

$= {}^{4}C_3 = {}^{4}C_1 = 4$ instead of 0

Hence ,actual triangles that can be drawn = $120 - 4 + 0 = 116$

06. there are 15 points in a plane out of which 5 are collinear . Prove that there are 445 triangles with vertices at these points

15 points

3 points define a triangle

$\therefore$ number of line that can be drawn $= {}^{15}C_3 = 455$

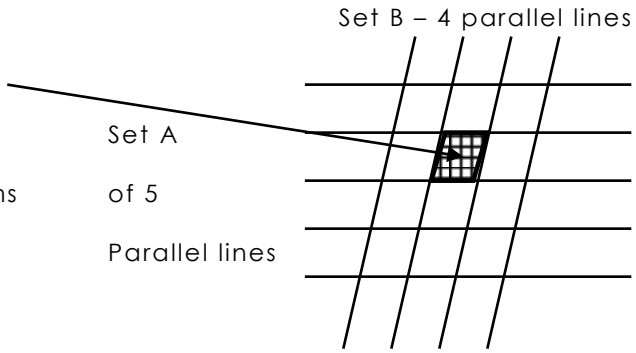
But 5 points are collinear

Number of triangles wrongly counted in these 5 collinear points

$= {}^{5}C_3 = {}^{5}C_2 = 10$ instead of 0

Hence actual triangles that can be drawn = $455 - 10 + 0 = 445$

09. Each of a set of 5 parallel lines cuts each one of another set of 4 parallel lines . How many different parallelograms can be formed **ans : 60**



2 lines from set A and 2 lines from set B define a parallelogram

therefore number of parallelograms that can be drawn

$= {}^5C_2 \times {}^4C_2$

$= 10 \times 6 = 60$

10. at the end of meeting , everyone had shaken hands with every one else . It was found that 45 handshakes were exchanged . How many members were present at the meeting .

'n' be the number of persons in the meeting

2 persons make a handshake

$\therefore$ number of handshakes $= {}^nC_2 = 45$ ..... Given

$$\frac{n(n-1)}{2} = 45$$

$$n(n-1) = 90$$

$n(n-1) = 10.9$ $\therefore$ On Comparison $n = 10$

EXTRAS

01. in how many ways can a man invite 6 friends to a party so that 2 or more of them remain present . **ans : 57**

man can invite 2 , 3 , 4 , 5 OR 6 friends

By Fundamental Principle of ADDITION

This can be done in $= {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6$

$= {}^6C_2 + {}^6C_3 + {}^6C_2 + {}^6C_1 + {}^6C_6$

$= 15 + 20 + 15 + 6 + 1$

$= 57$ ways

02. there are 5 questions in a question paper . In how many ways can a boy solve one or more questions .

boy can solve 1 , 2 , 3 , 4 , OR all 5 questions

By Fundamental Principle of ADDITION

This can be done in $= {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5$

$= {}^5C_1 + {}^5C_2 + {}^5C_2 + {}^5C_1 + {}^5C_5$

$= 5 + 10 + 10 + 5 + 1 = 31$ ways

03. in order to pass the examination a minimum is to be secured in each of the 7 subjects . In how many ways can a student fail. **ans : 127**

student can fail by securing less than minimum marks in 1 , 2 , 3 , 4 , 5 , 6 OR all 7 subjects

By Fundamental Principle of ADDITION

This can be done in $= {}^{7}C_1 + {}^{7}C_2 + {}^{7}C_3 + {}^{7}C_4 + {}^{7}C_5 + {}^{7}C_6 + {}^{7}C_7$

$= {}^{7}C_1 + {}^{7}C_2 + {}^{7}C_3 + {}^{7}C_3 + {}^{7}C_2 + {}^{7}C_1 + {}^{7}C_7$

$= 7 + 21 + 35 + 35 + 21 + 7 + 1$

$= 127$ ways

COMBINATION – SOLUTION TO Q SET-4

01. *${}^{n}C_{4} = 5\,{}^{n}P_{3}$, find n*

${}^{n}C_{4} = 5\,{}^{n}P_{3}$

$$\frac{\cancel{n!}}{(n-4)!\,.\,4!} = 5.\frac{\cancel{n!}}{(n-3)!}$$

$$\frac{(n-3)!}{(n-4)!} = 5.4!$$

$$\frac{(n-3)\,(n-4)!}{(n-4)!} = 5.4!$$

$$(n-3) = 5(24)$$

$$n-3 = 120$$

$$n = 123$$

01. *${}^{n}P_{r} = 120$ & ${}^{n}C_{r} = 20$, find n and r*

${}^{n}P_{r} = {}^{n}C_{r}\,.\,r!$

$120 = 20\,.\,r!$

$r! = 6$

$r! = 3!$

$r = 3$

Now ;

${}^{n}P_{r} = 120$

${}^{n}P_{3} = 120$

$$\frac{n!}{(n-3)!} = 120$$

$$\frac{n(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} = 120$$

$$n(n-1)(n-2) = 120$$

$$n(n-1)(n-2) = 6.5.4$$

On Comparing ; $n = 6$

02. *${}^{n}P_{r} = 720$ & ${}^{n}C_{r} = 20$, find n and r*

${}^{n}P_{r} = {}^{n}C_{r}\,.\,r!$

$720 = 120\,.\,r!$

$r! = 6$

$r! = 3!$

$r = 3$

Now ;

${}^{n}P_{r} = 720$

${}^{n}P_{3} = 720$

$$\frac{n!}{(n-3)!} = 720$$

$$\frac{n(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} = 720$$

$$n(n-1)(n-2) = 720$$

$$n(n-1)(n-2) = 10.9.8$$

On Comparing ; $n = 10$

03. *${}^{n}C_{6} : {}^{n-3}C_{3} = 33 : 4$, find n*

$$\frac{{}^{n}C_{6}}{{}^{n-3}C_{3}} = \frac{33}{4}$$

$$\frac{\dfrac{n!}{(n-6)!.6!}}{\dfrac{(n-3)!}{(n-3-3)!.3!}} = \frac{33}{4}$$

$$\frac{\frac{n!}{(n-6)!.6!}}{\frac{(n-3)!}{(n-6)!.3!}} = \frac{33}{4}$$

$$\frac{n!}{6!} \times \frac{3!}{(n-3)!} = \frac{33}{4}$$

$$\frac{n!}{(n-3)!} \times \frac{3!}{6!} = \frac{33}{4}$$

$$\frac{n(n-1)(n-2)(n-3)!}{(n-3)!} \times \frac{3!}{6.5.4.3!} = \frac{33}{4}$$

$$\frac{n(n-1)(n-2)}{6.5.4} = \frac{33}{4}$$

$$n(n-1)(n-2) = 33 \,.\, 6 \,.5$$

$$n(n-1)(n-2) = 990$$

$$n(n-1)(n-2) = 11 \times 10 \times 9$$

On Comparing ; n = 11

04. *$^{14}C_{2r} : {}^{10}C_{2r-4} = 143 : 10$, find r*

$$\frac{{}^{14}C_{2r}}{{}^{10}C_{2r-4}} = \frac{143}{10}$$

$$\frac{\frac{14!}{(14-2r)!\,.\,2r\,!}}{\frac{10\,!}{(10-2r+4)!\,.\,(2r-4)!}} = \frac{143}{10}$$

$$\frac{\frac{14!}{(14-2r)!\,.\,2r\,!}}{\frac{10\,!}{(14-2r)!\,.\,(2r-4)!}} = \frac{143}{10}$$

$$\frac{14!}{2r\,!} \times \frac{(2r-4)!}{10!} = \frac{143}{10}$$

$$\frac{(2r-4)!}{2r!} \times \frac{14!}{10!} = \frac{143}{10}$$

$$\frac{(2r-4)!}{2r\,.(2r-1)(2r-2)(2r-3)(2r-4)!} \times \frac{14.13.12.11.10!}{10!} = \frac{143}{10}$$

$$\frac{14.13.12.11}{2r\,.(2r-1)(2r-2)(2r-3)} = \frac{143}{10}$$

$$2r\,.(2r-1)(2r-2)(2r-3) = 14\,.\,12\,.\,10$$

$$2r\,.(2r-1)(2r-2)(2r-3) = 7.2\,.\,6.2\,.\,5.2$$

$$2r\,.(2r-1)(2r-2)(2r-3) = 8\,.\,7\,.\,6\,.\,5$$

On comparing ; $2r = 8 \;\therefore\; r = 4$

05. *$^{28}C_{2r} : {}^{24}C_{2r-4} = 225 : 11$, find r*

$$\frac{{}^{28}C_{2r}}{{}^{24}C_{2r-4}} = \frac{225}{11}$$

$$\frac{\frac{28!}{(28-2r)!\,.\,2r\,!}}{\frac{24\,!}{(24-2r+4)!\,.\,(2r-4)!}} = \frac{225}{11}$$

$$\frac{\frac{28!}{(28-2r)!\,.\,2r\,!}}{\frac{24\,!}{(28-2r)!\,.\,(2r-4)!}} = \frac{225}{11}$$

$$\frac{28!}{2r\,!} \times \frac{(2r-4)!}{24!} = \frac{225}{11}$$

$$\frac{(2r-4)!}{2r!} \times \frac{28!}{24!} = \frac{225}{11}$$

$$\frac{(2r-4)!}{2r\,.(2r-1)(2r-2)(2r-3)(2r-4)!} \times \frac{28.27.26.25.24!}{24!} = \frac{225}{11}$$

$$\frac{28.\cancel{27}^{3}.26.\cancel{25}^{9}}{2r\,.(2r-1)(2r-2)(2r-3)} = \frac{\cancel{225}}{11}$$

$$2r\,.(2r-1)(2r-2)(2r-3) = 28\,.\,3\,.\,26\,.\,11$$

$$2r\,.(2r-1)(2r-2)(2r-3) = 14.2.3.13.2.11$$

$$2r\,.(2r-1)(2r-2)(2r-3) = 14.13.12.11$$

On comparing ; $2r = 14 \quad \therefore\ r = 7$

06. *$^{10}C_{r+2} : {}^{10}C_r = 10 : 21$, find r*

$$\frac{{}^{10}C_{r+2}}{{}^{10}C_r} = \frac{10}{21}$$

$$\frac{\frac{10!}{(10-r-2)!\,(r+2)!}}{\frac{10!}{(10-r)!\,.\,r!}} = \frac{10}{21}$$

$$\frac{(10-r)!\,.\,r!}{(8-r)!\,.\,(r+2)!} = \frac{10}{21}$$

$$\frac{(10-r)(9-r)(8-r)!}{(8-r)!}\,\frac{r!}{(r+2)(r+1)\,r!} = \frac{10}{21}$$

$$\frac{(10-r)(9-r)}{(r+2)(r+1)} = \frac{10}{21}$$

$$\frac{90-10r-9r+r^2}{r^2+2r+r+2} = \frac{10}{21}$$

$$\frac{90-19r+r^2}{r^2+3r+2} = \frac{10}{21}$$

$$1890-399r+21r^2 = 10r^2+30r+20$$

$$11r^2-429r+1870 = 0$$

$$r^2-39r+170 = 0$$

$$(r-34)(r-5) = 0$$

$$r \neq 34 \text{ or } r = 5$$

07. *$^{n}C_{r-1} : {}^{n}C_r : {}^{n}C_{r+1}$*

$= 20 : 35 : 42$, find n & r

$$\frac{{}^{n}C_{r-1}}{{}^{n}C_r} = \frac{20}{35}$$

$$\frac{\frac{\cancel{n!}}{(n-r+1)!\,.\,(r-1)!}}{\frac{\cancel{n!}}{(n-r)!.r!}} = \frac{20}{35}$$

$$\frac{(n-r)!\,.\,r!}{(n-r+1)!\,.\,(r-1)!} = \frac{20}{35}$$

$$\frac{(n-r)!}{(n-r+1)!\,.\,(r-1)!}\,\frac{r!}{} = \frac{20}{35}$$

$$\frac{\cancel{(n-r)!}}{(n-r+1).\cancel{(n-r)!}}\,\frac{r\,.\,\cancel{(r-1)!}}{\cancel{(r-1)!}} = \frac{20}{35}$$

$$\frac{r}{n-r+1} = \frac{4}{7}$$

$$7r = 4n-4r+4$$

$$11r-4n = 4 \quad \ldots\ldots\ldots \quad (1)$$

Now

$$\frac{{}^{n}C_r}{{}^{n}C_{r+1}} = \frac{35}{42}$$

$$\frac{\dfrac{n!}{(n-r)!\,.\,r!}}{\dfrac{n!}{(n-r-1)!.(r+1)!}} = \frac{35}{42}$$

$$\frac{(n-r-1)!\,.\,(r+1)!}{(n-r)!\,.\,r!} = \frac{35}{42}$$

$$\frac{(n-r-1)!}{(n-r)!\,.}\ \frac{(r+1)!}{r!} = \frac{35}{42}$$

$$\frac{(n-r-1)!}{(n-r).(n-r-1)!}\ \frac{(r+1).r!}{r!} = \frac{35}{42}$$

$$\frac{r+1}{n-r} = \frac{5}{6}$$

$$6r + 6 = 5n - 5r$$

$$11r - 5n = -6 \quad \ldots\ldots\ldots \quad (2)$$

Solving (1) & (2)

$$11r - 4n = 4$$

$$11r - 5n = -6$$

$$- \quad + \quad +$$

$$n = 10$$

subs in 1 : $11r - 40 = 4$

$$11r = 44$$

$$r = 4$$

08. *$^nC_{r-1} = 495$; $^nC_r = 220$; $^nC_{r+1} = 66$, find n & r*

$$\frac{^nC_{r-1}}{^nC_r} = \frac{495}{220}$$

$$\frac{\dfrac{n!}{(n-r+1)!\,.\,(r-1)!}}{\dfrac{n!}{(n-r)!.r!}} = \frac{9}{4}$$

$$\frac{(n-r)!\,.\,r!}{(n-r+1)!\,.\,(r-1)!} = \frac{9}{4}$$

$$\frac{(n-r)!}{(n-r+1)!\,.}\ \frac{r!}{(r-1)!} = \frac{9}{4}$$

$$\frac{(n-r)!}{(n-r+1).(n-r)!}\ \frac{r\,.\,(r-1)!}{(r-1)!} = \frac{9}{4}$$

$$\frac{r}{n-r+1} = \frac{9}{4}$$

$$4r = 9n - 9r + 9$$

$$13r - 9n = 9 \quad \ldots\ldots\ldots \quad (1)$$

Now

$$\frac{^nC_r}{^nC_{r+1}} = \frac{35}{42}$$

$$\frac{\dfrac{n!}{(n-r)!\,.\,r!}}{\dfrac{n!}{(n-r-1)!.(r+1)!}} = \frac{220}{66}$$

$$\frac{(n-r-1)!\,.\,(r+1)!}{(n-r)!\,.\,r!} = \frac{10}{3}$$

$$\frac{(n-r-1)!}{(n-r)!\,.}\ \frac{(r+1)!}{r!} = \frac{10}{3}$$

$$\frac{(n-r-1)!}{(n-r).(n-r-1)!}\ \frac{(r+1).r!}{r!} = \frac{10}{3}$$

$$\frac{r+1}{n-r} = \frac{10}{3}$$

$$3r + 3 = 10n - 10r$$

$$13r - 10n = -3 \quad \ldots\ldots\ldots \quad (2)$$

Solving (1) & (2)

$$13r - 9n = 9$$

$$13r - 10n = -3$$

$$- \quad + \quad +$$

$$n = 12 \ , r = 9$$

09. ${}^{14}C_5 + {}^{14}C_6 + {}^{15}C_7 + {}^{16}C_8 = {}^{17}C_x$, find x

Using ${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$

${}^{14}C_5 + {}^{14}C_6 + {}^{15}C_7 + {}^{16}C_8 = {}^{17}C_x$,

${}^{15}C_6 + {}^{15}C_7 + {}^{16}C_8 = {}^{17}C_x$,

${}^{16}C_7 + {}^{16}C_8 = {}^{17}C_x$,

${}^{17}C_8 = {}^{17}C_x$,

${}^{17}C_8 = {}^{17}C_9 = {}^{17}C_x$ $\therefore$ x = 8 OR 9

10. ${}^{25}C_4 + {}^{25}C_5 + {}^{26}C_6 + {}^{27}C_7 = {}^{28}C_r$, find r

Using ${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$

${}^{25}C_4 + {}^{25}C_5 + {}^{26}C_6 + {}^{27}C_7 = {}^{28}C_r$,

${}^{26}C_5 + {}^{26}C_6 + {}^{27}C_7 = {}^{28}C_x$,

${}^{27}C_6 + {}^{27}C_7 = {}^{28}C_x$,

${}^{28}C_7 = {}^{28}C_x$,

${}^{28}C_7 = {}^{28}C_{21} = {}^{28}C_x$ $\therefore$ x = 7 OR 21

11. ${}^{12}C_5 + 2.{}^{12}C_4 + {}^{12}C_3 = {}^{14}C_x$,

find x

${}^{12}C_5 + {}^{12}C_4 + {}^{12}C_4 + {}^{12}C_3 = {}^{14}C_x$

${}^{13}C_5 + {}^{13}C_4 = {}^{14}C_x$

${}^{14}C_5 = {}^{14}C_x$

Now ; ${}^{14}C_5 = {}^{14}C_9 = {}^{14}C_x$

x = 5 OR 9

12. ${}^{47}C_4 + \sum_{r=1}^{5} {}^{52-r}C_3$.

$= {}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{47}C_4$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{48}C_4$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{49}C_4$

$= {}^{51}C_3 + {}^{50}C_3 + {}^{50}C_4$

$= {}^{51}C_3 + {}^{51}C_4$

$= {}^{52}C_4$

Hence ;

$${}^{47}C_4 + \sum_{r=1}^{5} {}^{52-r}C_3 = {}^{52}C_4$$

13. find the difference between the greatest values of $^{13}C_r$ & $^{11}C_r$

STEP 1 :

the greatest value of $^{13}C_r$ occurs at

$$r = \frac{n-1}{2} = \frac{13-1}{2} = 6$$

Hence the greatest value of $^{13}C_r$

$$= {}^{13}C_6$$

$$= \frac{13!}{(13-6)!\,.\,6!}$$

$$= \frac{13!}{7!.6!}$$

$$= \frac{13 \times 12 \times 11 \times 10 \times 9 \times 8}{6 \times 5 \times 4 \times 3 \times 2 \times 1} = 1716$$

STEP 2 :

the greatest value of $^{11}C_r$ occurs at

$$r = \frac{n-1}{2} = \frac{11-1}{2} = 5$$

Hence the greatest value of $^{11}C_r$

$$= {}^{11}C_5$$

$$= \frac{11!}{(11-5)!\,.\,5!}$$

$$= \frac{11!}{6!.5!}$$

$$= \frac{11 \times 10 \times 9 \times 8 \times 7}{5 \times 4 \times 3 \times 2 \times 1} = 462$$

STEP 3 :

Difference = 1716 – 462 = 1252

14. find the difference between the greatest values of $^{14}C_r$ & $^{8}C_r$

STEP 1 :

the greatest value of $^{14}C_r$ occurs at

$$r = \frac{n}{2} = \frac{14}{2} = 7$$

Hence the greatest value of $^{14}C_r$

$$= {}^{14}C_7$$

$$= \frac{14!}{(14-7)!\,.\,7!}$$

$$= \frac{14!}{7!.7!}$$

$$= \frac{14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} = 3432$$

STEP 2 :

the greatest value of $^{8}C_r$ occurs at

$$r = \frac{n}{2} = \frac{8}{2} = 4$$

Hence the greatest value of $^{8}C_r$

$$= {}^{8}C_4$$

$$= \frac{8!}{(8-4)!\,.\,4!}$$

$$= \frac{8!}{4!.4!}$$

$$= \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} = 70$$

STEP 3 :

Difference = 3432 – 70 = 3362

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15. find the difference between the greatest values of $^{15}C_r$ & $^{12}C_r$

STEP 1 :

the greatest value of $^{15}C_r$ occurs at

$$r = \frac{n-1}{2} = \frac{15-1}{2} = 7$$

Hence the greatest value of $^{15}C_r$

$$= {}^{15}C_7$$

$$= \frac{15!}{(15-7)!\,.\,7!}$$

$$= \frac{15!}{8!.7!}$$

$$= \frac{15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} = 6435$$

STEP 2 :

the greatest value of $^{12}C_r$ occurs at

$$r = \frac{n}{2} = \frac{12}{2} = 6$$

Hence the greatest value of $^{12}C_r$

$$= {}^{12}C_6$$

$$= \frac{12!}{(12-6)!\,.\,6!}$$

$$= \frac{12!}{6!.6!}$$

$$= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7}{6 \times 5 \times 4 \times 3 \times 2 \times 1} = 924$$

STEP 3 :

Difference = 6435 – 924 = 5511